

### FST 2-4 Notes

Topic: Exponential Functions

GOAL:

Review the properties of exponential functions.

#### SPUR Objectives

**E** Describe properties of exponential functions.

### Vocabulary

growth factor

exponential function with base  $b$

exponential growth function

exponential growth curve

exponential decay function

asymptote

### Warm up

Suppose you earn \$1000 on a job and 10% is taken out for income tax. Then your employer gives you a 10% raise. How much will you have after the raise and tax?

$$1000(1 - .10) = 900$$
$$900(1 + .10) = 990$$

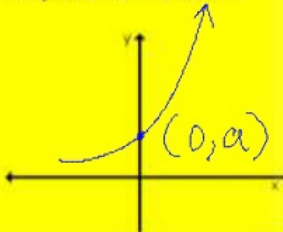
### Definition of Exponential Function

An **exponential function with base  $b$**  and initial value  $a$  is a function with an equation of the form

$$f(x) = ab^x,$$

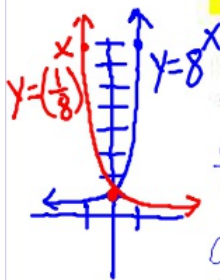
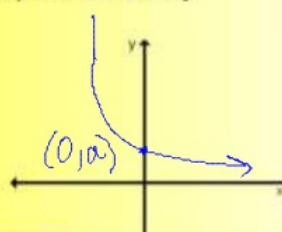
where  $a \neq 0$ ,  $b > 0$ , and  $b \neq 1$ .

Exponential Growth



$$y = a \cdot b^x$$

Exponential Decay



Compare and contrast the graphs of the three functions  $f$ ,  $g$ , and  $h$ , where  $f(x) = 8^x$ ,  $g(x) = \left(\frac{1}{8}\right)^x$ , and  $h(x) = 8^{-x}$  for all values of  $x$ .

$f(x) = 8^x$   $b=8$  ( $b > 1$ ) exponential growth

$g(x) = \left(\frac{1}{8}\right)^x$   $b=\frac{1}{8}$  ( $0 < b < 1$ ) exponential decay

$h(x) = 8^{-x} = \frac{1}{8^x} = \left(\frac{1}{8}\right)^x = g(x)$  exponential decay

Domain:  $\{x \mid x \in \mathbb{R}\}$  Range:  $\{y \mid y > 0\}$

1. Presently, the towns of Scarcedale and Ampleton both have approximately 8500 residents. Over the next 5 years, the population of Ampleton is expected to increase by approximately 2.3% per year, while the population of Scarcedale is expected to decrease by about 0.9% each year.
- Create equations to describe the population of each town as a function of time.
  - Compare the projected populations after 5 years.

Ampleton  $\uparrow$  2.3%

$$(1 + .023)$$

Scarcedale  $\downarrow$  0.9%

$$(1 - .009)$$

$$y = a \cdot b^x$$

$y = (\text{start value}) (\text{growth/decay factor})^{\text{time}}$

a)  $y = 8500(1 + .023)^t$

b)  $y = 8500(1.023)^5$

$y = 9524$

a)  $y = 8500(1 - .009)^t$

b)  $y = 8500(.991)^5$

$y = 8124$

★ population differs by 1400 in 5 yrs

2. Jonas received a letter from his credit union stating that a 5-year CD his parents opened for him had matured and he could choose one of the following options:

- (1) Withdraw the full amount of \$14,204.10.
- (2) Use the money to open a new 5-year CD with an APY of 2.81%.
- (3) Have the money deposited into his interest checking account which earns 0.95% per year.

$1 + 9\%$   
 $2.81\%$   
 $.0281$

Assume Jonas's parents deposited \$12,000 into the CD 5 years ago.

- a. What is an equation for the balance of the old CD after 5 years?
- b. Use the equation from Part a to find the annual yield of the old CD.
- c. Compare the results of options (2) and (3).

a)  $A = P(1+r)^t$

$A =$  balance after  $t$  years

$P =$  principle (beginning amount)

$t =$  years

$r =$  APY (Annual Percentage Yield)

c)  $A = P(1+r)^t$   
 $= 14,204.10(1+.0281)^5$

$= \$16,315.13$

$= 14,204.10(1+.0095)^5$

$= \$14,891.74$

a)  $14,204.10 = 12,000(1+r)^5$

$\frac{14,204.10}{12,000} = \frac{12,000}{12,000}(1+r)^5$

$\sqrt[5]{1.183675} = \sqrt[5]{(1+r)^5}$

$1.0343 = 1+r$

$.0343 = r$   
 $\times 100$

$3.43\%$